ThinQ
think • inquire • question

CHAPTER 1
INVITATION TO INQUIRY

We begin our journey with a story about visitors to our planet. Let us hope this journey will be an exciting and challenging one.

1.1 Scientific Inquiry: A Taroan learns Magomese

A Story

Everyone knows that Taroans (beings from planet Taro) communicate with one another directly through brain waves. They don't need a language mediated through speech sounds or writing to communicate.

On 15 August 2000, a Taroan called Zebluni (Zee for short) came to Earth for a holiday and landed in a village called Magomi in a small country called Luga. On the very first day, she discovered that the humans of Luga kept making noises with their mouth when they were with another human. Because of her capacity to interpret brain waves, she discovered that when Magomians made these noises with their mouth, they were communicating a message, which other Magomians understood. She was puzzled about why they were making these strange noises when communicating.

By the end of the day, she had a HYPOTHESIS:

> *Magomians communicate through sounds they make with their mouth.*
> *They don't know how to communicate through brain waves.*

She decided to test her hypothesis, and to do a research project on their mode of communication. To do this, she would need to gather DATA; so, she started recording the utterances that she heard, and also their meanings. This was easy for her because of her brainwave reading capacity.

By the end of the day, she was convinced that her hypothesis was correct. She figured out that a lot of what she had noted down in her logbook were units that Magomians called 'sentences'. She had by now documented a large number of sentences. Some of the sentences she recorded were:

1	snulabimisu	The cow kicked the boy
2	flantmisulabi	The boy tickled the cow
3	gumpimisuadofromp	The boy sent the girl a book
4	flantmisuado	The boy tickled the girl
5	flantadomisu	The girl tickled the boy
6	gumpiadomisuslapsir	The girl sent the boy a ball
7	brimadolabiblenk	The girl gave the cow a banana

After poring over her notes all evening, Zee now wrote down a new sentence, to see if she could translate it into Magomese. Here is the sentence she wrote, given in English. Can you translate it into Magomese?

The cow gave the boy a book.

How about translating the following Magomese sentence into English? Zee was able to translate it.

gumpilabiadoslapsir

Can you now list the Magomese words you have learnt, with their English meanings?

Do not read further till you have wrestled sufficiently with the tasks in the box above, whether or not you have been successful in cracking the code.

How are we able to perform these tasks? Reflect on the processes (mental strategies) that you used to do them.

What was the challenge in this activity? You must have realized that it is to identify the words of the language (Magomese) by breaking down each sentence into smaller parts, and then figure out which order of words yields which meaning.

To do this, a useful strategy is to take pairs of sentences that *differ minimally in meaning, with a corresponding difference in the sounds.* Take, for instance, the following sentences:

flantmisulabi 'the boy tickled the cow'

flantmisuado 'the boy tickled the girl'

As far as the meaning is concerned, the only difference is that the first sentence has 'the cow' and the second sentence has 'the girl'. The corresponding difference in Magomese is that the first sentence has *labi* while the second sentence has *ado*. Hence, it is likely that *labi* means 'the cow,' and *ado* means 'the girl'. We can now use these sequences of sounds in the other sentences to break up the sentence at least in part. Following this strategy, it should be possible to identify every word in the given data, with their meanings.

Once the words and their meanings are identified, we can figure out in which order the 'subject', the 'object', and the 'verb' occur in Magomi. What is the 'word order' in this language?

Is it Subject-Verb-Object as in English,
Subject-Object-Verb as in many Indian languages,
or some other order, such as:
Verb-Subject-Object as in Irish or Scottish.

This is the first step that a linguist has to follow when 'studying' a previously unexplored language. In going through this activity, we are looking for patterns in the observed expressions of a language. Such activities help us develop ***the capacity to detect patterns***.

In the tasks of translating from English to Magomese and from Magomese to English, and picking out the pieces to translate, notice that we took for granted two CONCEPTS, 'sentence' and 'word', without ***defining*** them. We have also mentioned other concepts like 'word order', 'subject', 'object', and 'verb'. Do we know what these concepts mean?

As we proceed, we will see the need to explicitly acknowledge such concepts, define them, and work out the consequences of our definitions and generalizations. Doing this in a systematic way is part of the strategy needed in mathematical inquiry, scientific inquiry (including historical inquiry), conceptual inquiry, and ethical inquiry.

> If you found this activity exciting, and would like to explore more such problems in scientific inquiry into Language Structure, you may wish to take a look at *Exploring Patterns in Language Structure*

1.2 Mathematical Inquiry can be Fun

Do you like math? Or are you a mathophobic? (Mathophobics are afraid of math and feel that they are not good at it.) If you are, maybe you found school math extremely frightening, confusing, or boring, and came away feeling that you are no good at it. This happens to a lot of people during their school years, even to those who actually have a high aptitude for mathematical thinking.

So, you shouldn't be discouraged. Perhaps you will even fall in love with math if you come face to face with real math! Let us go through some examples to give you a flavour of what you will find in this book. Suppose you found the following question in an entrance test:

Is the sum of 74 consecutive numbers divisible by 74?

[When we say 'number' in this question, we mean counting numbers like 1, 2, 3, 58, 470, and 9865. We are not including fractions like ½ and ¼, or decimals like 6.82. We are also excluding zero. And by 'divisible' we mean divisible without remainders, fractions, or decimals.]

Did you already know the answer? Did you try looking for one in a textbook or on the internet? If you answered yes to the second question, you are probably not alone. How about finding the answer on your own by thinking about it? You can find the answer to this question yourself if you adopt certain strategies of thinking that mathematicians use.

Let us explore one way of doing this. The question posed above, "Is the sum of *74* consecutive numbers divisible by *74*?" is of the following form:

> Is the sum of *n* consecutive numbers divisible by *n*?

Let us take some specific cases that are easier:

> Is the sum of 2 consecutive numbers divisible by 2?
>
> Is the sum of 3 consecutive numbers divisible by 3?
>
> Is the sum of 4 consecutive numbers divisible by 4?
>
> Is the sum of 5 consecutive numbers divisible by 5?

Try the first one.

1 + 2 = 3	(not divisible by 2)	4 + 5 = 9	(not divisible by 2)
2 + 3 = 5	(not divisible by 2)	13 + 14 = 27	(not divisible by 2)
3 + 4 = 7	(not divisible by 2)	227 + 228 = 455	(not divisible by 2)

It looks like the answer to this question is 'No.' Let us state our guess as follows:

> The sum of 2 consecutive numbers is not divisible by 2.

This statement is a mathematical CONJECTURE. Can you come up with a PROOF for this conjecture? Now, in mathematics, we can say that we have proved something only when we are absolutely certain about it.

In the pairs of consecutive numbers we looked at, we found that their sum is not divisible by 2. But suppose there is some example that we have missed, where the sum of two consecutive numbers is indeed divisible by 2. To be absolutely sure about our conjecture, we need to prove it such that no matter which two consecutive numbers we take, the sum is not divisible by two.

Before you read any further, spend some time to think. Then try writing down a proof, even if you cannot complete it.

When you have made your best attempt, do the exercise given below. (Do not go further till you have at least made a serious attempt.)

Draw a triangle, say, as in Figure 1.

Now measure the lengths of sides AB, AC, and BC.

Add the lengths of AB and AC, and you will find that BC is less than AB+AC.

Can we make BC longer than AB+AC? Let us try.

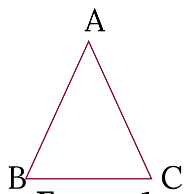


FIGURE 1

Extend BC to BD such that BD is at least twice BC, as in Figure 2.

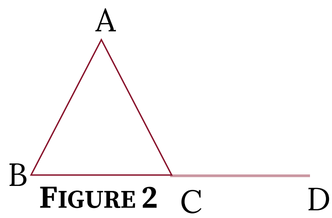


FIGURE 2

Then connect A to D, forming triangle ABD, as in Figure 3.

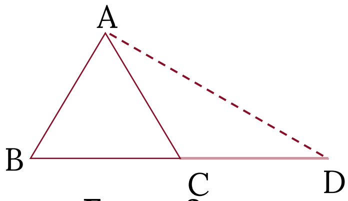


FIGURE 3

If we measure AB, AD and BD, and add up AB and AD, we will find that BD is shorter than AB+AD.

Try extending BD further, to BE, such that BE is four times, five times, six times, seven times longer than BD, and then connect A and E. Measure again. If BE is still shorter, make it really long, say ten or twenty times.
Measure again. Any luck?

Now, instead of making BC longer, how about making AB shorter?

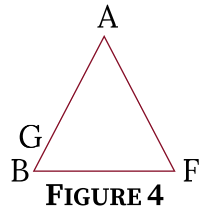


FIGURE 4

On triangle ABF, take a point G close to B, as in Figure 4.

Then connect G and F, as in Figure 5.
Now measure GB, GF and BF.

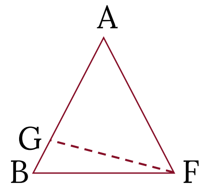


FIGURE 5

Is BF shorter than GB+GF? If it is, try making GB still shorter. Continue doing this till you cannot do it anymore.

What did you learn from this exercise?

Write it down in one sentence. Here is the first part; what you have to do is complete the sentence:

In any triangle, the length of any one side is ……………………………………………

What you have written down is a conjecture, a statement that you think is true. But you have not proved it yet. Can you prove it?

We want you to *try* to prove it, if not prove it. We will come back to this later and give you a hint to help you.

Before we got to the above exercise, our task was to prove the conjecture:

> The sum of 2 consecutive numbers is not divisible by 2.

Did you hit upon a proof? If your proof begins with a statement like:

> Let x be any number.

chances are that you already have a proof. Congratulations!

If you have not thought of: “Let x be any number,” perhaps you can try again, starting with that statement, and write the remaining part of the proof. Then go to the next page to look at our proof.

Proof: "The sum of 2 consecutive numbers is not divisible by 2."

Let x be any number.

The next number then is $x + 1$

The sum of the two numbers is $(x) + (x+1) = 2x + 1$

Dividing by 2, we get $\frac{(2x + 1)}{2} = x + \frac{1}{2}$

Therefore, $2x + 1$ is not divisible by 2.

Thus, whatever be the numbers, the sum of 2 consecutive numbers is not divisible by 2. *QED*

[QED stands for the Latin phrase *quod erat demonstrandum*, meaning, "that which is to be proved."]

We started with the conjecture:

The sum of 2 consecutive numbers is not divisible by 2.

Now that we have proved it, it has become a THEOREM.

We suggest that you try to answer the other questions — those about the divisibility of 3, 4, 5 … consecutive numbers by 3, 4, 5 … respectively — adopting the same strategy.

First, try to prove the conjectures that you propose as your answers. Having done that, ask yourself if you see a pattern in the theorems. If you see the pattern, write it down. Then go back to:

Is the sum of n consecutive numbers divisible by n?

If you can answer that question, you can also answer the question that we started with:

Is the sum of 74 consecutive numbers divisible by 74?

If you are afraid of math, this question may have frightened you. But having approached it with easier examples and gone through them systematically, you must have come up with an answer.

Notice that we have not given you answers to any of these questions; all we have done is give minimal guidance, such that *you* find the answers on your own. This was to show you that there are strategies that you can adopt to answer many questions on your own, without getting the answer from a textbook or the internet, or asking a teacher or someone else who might know the answer.

Let us now explicitly state the strategies of inquiry that we used above:

If we find that a mathematical (or any other) problem is too difficult or too complex, we

- try to ***unpack*** it into a set of easier or smaller problems;
- solve the easier/simpler ones first; and
- then go back to the more difficult ones.

In doing so, before confronting the original problem, we

(a) generalize specific problems; and

(b) find special instances of general problems.

The number in our initial question, 74, is a large one. So, using strategy (a), we **generalized** the question to:

"Is the sum of n consecutive numbers divisible by n?"

Using strategy (b), we moved to the easier task of examining smaller and more manageable numbers, like 2 and 3. We were then ready to go back to the original problem.

There is an important lesson to take away from the exercise above. Often, if we want to find the answer to a question, we do not need to ask a teacher, or look for it in a textbook or on the internet. We can find the answer on our own, by using **TOOLS OF INQUIRY**. What this book hopes to do is to introduce you to such a set of tools for knowledge building.

1.3 What you would Learn from this Journey

When we step into new territories of learning, a good way to start is to separate

what we already know, and

what we want to learn.

We have seen three examples of problems in such new territories, one on a new language, a second on numbers, and a third on shapes. They all required ***pattern-finding abilities***. The one on numbers also required the ability to break down a problem into simpler pieces to unearth a proof. The triangle problem was a little more difficult. It required identifying and formulating a pattern that one may not have even imagined before, and its proof is even less obvious.

Here are a few more interesting questions that you will come across later in this journey. If they bug you, and prompt you to look for answers, keep reading.

A. Textbooks tell us that a solid has its own shape and volume; a liquid has its own volume but takes the shape of its container; and a gas has neither definite volume nor definite shape. A handful of wheat and a piece of sewing thread have their own volume but take the shape of their container. Does this mean they are liquid? A soap bubble has its own spherical shape, and so does the sun. Are they solid? To figure out answers to these questions, you have to learn ***to critically evaluate definitions, as well as create them***.

B. Take this scenario. Miko is 13, and her brother Jomo is 11. They take the seeds of a chilli and plant them under a tree in their backyard. Only one seed sprouts and grows into a plant. When the plant bears fruit, and the chillies are ready to eat, Miko is disappointed because they are not hot. Why are the chillies not hot? Jomo thinks it is because the plants were not watered enough. Miko thinks it is because the plants did not get enough sun. Can you help them decide who is right?

   Can you design an experiment to resolve their dispute? For this, you will have to
   learn ***how to design experiments***.

C. Suppose you want to get home from the railway station. There are many different routes you can take. How will you find the fastest route? Notice, we have said *fastest* route, not *shortest* route. To find out, you will have to learn about ***variability and how to deal with it***.

D. To prove that the sum of 2 consecutive numbers is not divisible by 2, we had to show that there cannot exist a pair of consecutive numbers whose sum is divisible by 2. We used a particular kind of ***reasoning*** for this: the proof allowed no exception. Now take a different kind of example. Suppose we told you that Paro is an adult human being, and she has two hearts. Would you believe us? You will probably say that every human adult has one and only one heart. Can you prove this? It requires a different kind of reasoning. To understand the difference between the two kinds of proofs, you have to learn the differences between the ***ways in which scientists and mathematicians establish their findings***.

E. Imagine that among your friends, some believe that humans have souls, and others believe that souls do not exist. How would the two groups resolve their conflict of beliefs? For this, you have to learn ***to debate, and become proficient at it***.

If these questions arouse your interest, this book is for you. Welcome to the world of observing, thinking, inquiring, questioning!

In the coming chapters, we will learn to observe the world around us and within us, and to identify and formulate worthwhile questions that need exploring. We will look for answers to these questions and arrive at conclusions based on our answers. We will learn to critically evaluate the conclusions that we ourselves or others have arrived at; figure out their merit; give justification for our conclusions such that others are convinced; and engage in academic debates.

Before we end the chapter, we would like to introduce you to the concepts of 'rational inquiry', and 'academic intelligence'. They are both central to our exploration together.

1.4 What is Rational Inquiry?

This book is called 'Introduction to Rational Inquiry'. You might wonder what 'inquiry' is. Put simply, **INQUIRY** is the investigation of a question, relying on our own experience, observation, thinking, reasoning, and judgment, to look for an answer and arrive at a conclusion. We inquire because of our desire to find out something we do not know, or do not understand.

The word 'RATIONAL' means 'in accordance with reason.' Therefore, RATIONAL INQUIRY is inquiry that is in accordance with reason. Its components are:

- *questions* whose answers we wish to find out;
- *ways* to look for answers;
- *answers* to the questions, and *conclusions* based on them;
- *rational justification* (proof, evidence, arguments) for the conclusions; and
- *thinking critically* about conclusions and justification (of others as well as our own).

Rational inquiry is committed to the following axioms:

- ***Rejecting* LOGICAL CONTRADICTIONS***:* We must reject statements that are logically contradictory.
- ***Accepting* LOGICAL CONSEQUENCES***:* If we accept a set of statements, then we must also accept their logical consequences.

By *logical contradiction*, we mean a combination of a statement and its negation. Thus, the statement that the earth is flat, and the earth is not flat, contains a logical contradiction. A *logical consequence* of a set of statements is a conclusion derived from them through logic. For example:

given the statements:	(i) all humans are primates;
	(ii) all primates are mammals; and
	(iii) all mammals are vertebrates;
we would conclude that:	(iv) all humans are vertebrates.

The statement in (iv) is a logical consequence of statements (i)-(iii).

Many of the concepts and ideas we are talking about here may be entirely new to you, and hence confusing. You shouldn't worry; we will come back to them with plenty of examples.

1.5 Academic Intelligence

The knowledge that students are exposed to in school and college, under subject headings like mathematics, science, philosophy, and so on, is ACADEMIC KNOWLEDGE. Creating and evaluating academic knowledge requires the practice of rational inquiry. Such practice helps us develop ACADEMIC INTELLIGENCE. This book is an invitation to make your mind stronger, sharper, faster, fitter, and more agile through the pursuit of rational inquiry towards academic intelligence.

The strands of academic intelligence, forming a "syllabus" of abilities, might read like A-N below.

Academic intelligence includes the ability to:

A. systematically observe and describe what one observes;

B. design and conduct experiments;

C. notice and state patterns in a sample based on A and B;

D. establish observational generalizations based on C;

E. construct theories to explain the generalizations in D;

F. infer logical consequences, and detect logical contradictions;

G. classify, and evaluate classificatory systems;

H. define concepts, and evaluate definitions;

I. come up with and evaluate axioms;

J. unearth and evaluate hidden assumptions;

K. pursue step-by-step reasoning;

L. prove and refute claims;

M. think and read/listen critically; and

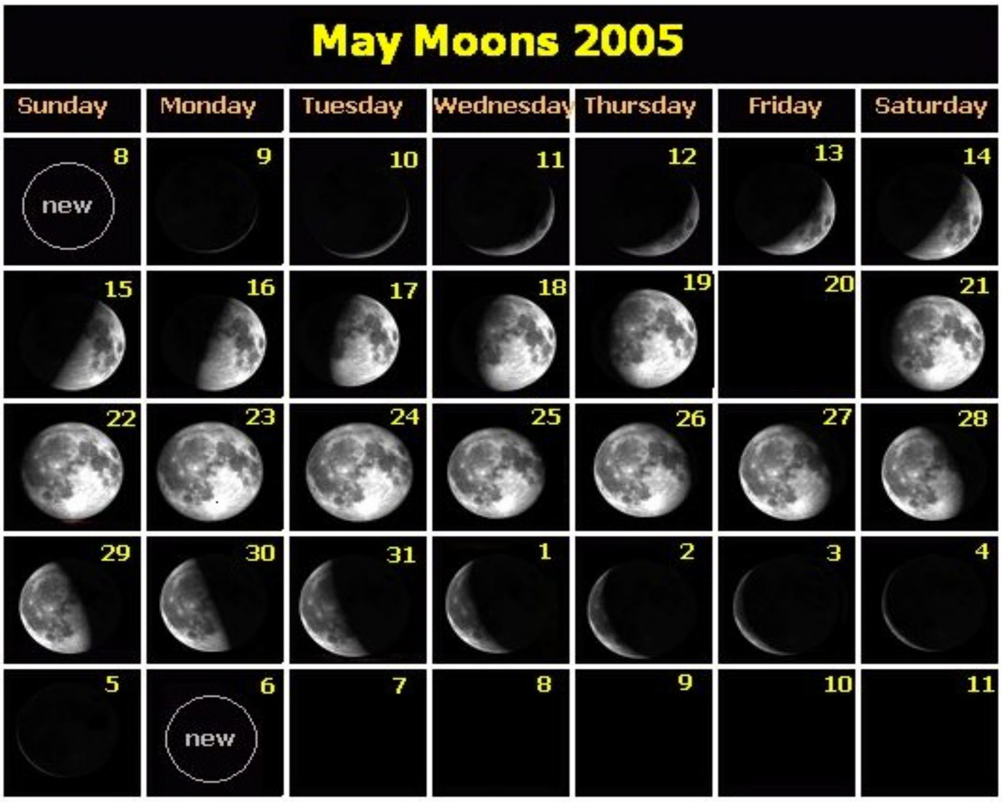
N. communicate with clarity, precision, and effectiveness.

You may not understand much of this list right now. But by the time you have finished with this book, we hope you will have a much better understanding.

Academic intelligence also acts as the foundation for developing discipline-specific research skills for those who want to pursue research. Like any other form of intelligence, including emotional intelligence and design intelligence, academic intelligence can be enhanced through guided practice, and is valuable to us all in our professional, public, and personal lives.

An Exercise

Consider the following chart, with different shapes of what we call the 'moon'.



[Image: Tomruen at English Wikipedia – Transferred from en.wikipedia to Commons. Available at: https://en.wikipedia.org/wiki/Lunar_phase#/media/File:Moon_phase_calendar_May2005.jpg]

Now take the statement:

> *The different shapes that we call the 'moon' are different appearances of the same object; they are not different objects.*

TASK 1: Construct an argument to defend the above statement. [Take, for instance, the photographs of "May Moons 2005" given above. Take the shapes you see on 11 May, 15 May, and 23 May. Are they the shapes of different objects, each object appearing on a different day? You will have to argue that this couldn't be the case.]

TASK 2: Argue that the 'single moon' you defended under Task 1 is spherical, and not a flat circular disk.

In doing Tasks 1 and 2, assume that (i) you are living in the Egypt, Greece, or India of ancient times, when there were no spaceships, or even telescopes; but that (ii) you accepted the idea that the moon does not emit its own light; it reflects the sun's light.

Further to this chapter, we recommend *Reading B: What is a Solid?*